Published as:
Smith, L. (2002). Piaget's model. In U. Goswami (ed) *Blackwell handbook of childhood cognitive development*. Oxford: Blackwell.

Piaget's Model

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PIAGET'S MODEL

Jean Piaget

Piaget (1896-1980) was born in Neuchâtel where he gained his PhD in biology in 1918. This year marked a turning-point. His first book *Recherche* (*Search*) led to his "psychological turn", in Zurich with Bleuler, in Paris with Simon, and in Geneva with Claparède. He held chairs at Neuchatel in 1925, at Geneva in 1929 until his retirement, and at the Sorbonne from 1952. Piaget gained his first honorary degree from Harvard in 1936, an APA Distinguished Scientist award in 1969, and the Erasmus Prize in 1972. He was Director of UNESCO's International Bureau of Education during 1929-67, and the founding Director of his international centre from 1955 until his death. [1]

Piaget's work commanded massive international attention throughout the last century - 'the most criticised author in the history of psychology, and I came through alive' was how he put it (in Smith, 1996:vi). Beilin (1992) was nearer the mark in comparing Piaget's contribution to that of Shakespeare on English. Other developmentalists are not sure about their legacy as 'heirs to the house that Jean built' (Scholnick, 1999). A different analogy is to view Piaget's project as the construction of a Grand Tower (Smith, 1993:vi). This project was not completed due to disputes in a tower of Babel and, instead, we have many fine small buildings - but no Grand Tower. What, then, should we do in the twenty-first century with the master-plan, Piaget's (1918:148) 'research-programme'? The open question is the extent to which its principles can be preserved in the re-design of the project. After all, its origin is plain to see as the problem of knowledge in philosophy. Piaget's brilliant advance was equally plain in his first book. Kant had asked a normative question "How is knowledge possible?". Piaget converted this into an empirical counter-part "How does knowledge develop?". This is a massive problem-shift. It secures a bridge-head between epistemology and psychology. And it generalises nicely to other norms and facts. In this context, "Piaget's theory" is the best available theory. [3] At any rate, that's my view, and what follows is its rationale.

Nature, nurture or *tertium quid*?

A classic problem in psychology concerns the contributions of nature and nurture to intellectual development. Vygotsky (1994:57) characterised these as the two "lines of development", heredity and culture, whose interaction merited a better characterisation. But two is two, two lines with a subtle interaction still to be clarified (Moll, 1994). Piaget (1918) never accepted this view. His stance required a further factor which acts between the "two lines". This is Piaget's (1923) *tertium quid* or third alternative.

One argument is biological. 'Amoeba, sponges, fish, and mammals transmit all their characteristics (and this is) a truly hereditary transmission; but they also transmit quite equally the most general properties of life in virtue of organisation, and that is not transmission in the same sense. (This is because) at every step of hereditary transmission, a *living* organisation is present as the necessary condition of particular transmissions since it determines the *activities* arising in that transmission (Piaget, 1971:323*) [4] This argument is based on a distinction between particular properties of an organism and general properties of organisation. Particular properties of an organism (eg: eye colour) are due to hereditary transmission. General properties of organisation (eg: classification

abilities) are due to something else as well. Firstly, classification is a property of an organism's activities, not an organism. Classifying a pack of fifty-two playing-cards may be dependent on visual search and motor coordination, but it is an agent – not the eye or hand – who does this. Secondly, classification is due to an organisation which can be manifest in different ways (eg: cards in two colours or four suits). This is not a biological difference. Thirdly, organising principles behind any classification may themselves change, and change for the better, even resulting in a true classification. The equality (eg: cards in two colours = cards in four suits) is a true equality. But truth is not a biological property. The advance from a (correct) classification to true knowledge is a major advance, which goes beyond biology. So the insufficiency of heredity is one reason for Piaget's *tertium quid* (cf. Goodwin, 1982; Messerly, 1996).

Piaget's social argument is a thought-experiment, also discussed by Vygotsky. Imagine a society of exact contemporaries whose members are the same age (for example, children aged seven years). This society has neither a traditional culture nor generational legacies from the past, still less older members. What would intellectual development be like in such a society? Vygotsky (1994:351) ruled out intellectual progress due to the absence of cultural tools and more advanced members to guide social participation. Piaget (1995a: 57) agreed that children would be deprived of 'the essential instrument of transmission'. In normal development, later generations gain benefits in that 'ideas which have been painfully "invented" by the greatest geniuses become, not merely accessible, but even easy and obvious, to schoolchildren' (p. 37). Even so, Piaget did not rule out development altogether. Although such a society would be disadvantageous, it would have compensations. One is the elimination of adult constraint (p.149). Social rules are typically interpreted as directives which lay down "this is how we do it, so this is how it ought to be done". This is 'normative pressure' (von Wright, 1983:55). Such pressure would be reduced for these children who would still have the task of devising their own social rules and of creating better ones. Crucially, this society would still pose an equivalent version of the same (normative) problem for children. That problem is the reconciliation of *son moi et al loi*, the self and normative law (1995a:241). This problem is general. Rules and practices in all domains and contexts have normative properties whose understanding is central to intellectual development.

The upshot is that, in Piaget's model, factors in nature and nurture make a necessary, but insufficient contribution to intellectual development. This conclusion has independent support (Richardson, 1998; Wachs, 2000). Second, there is a third factor which is activated inter-dependently with the other two factors. [5] This is equilibration. This is the central construct (1918:46; 1923:57; 1950a:36; 1970:120; 1971:355; 1985:3; 1995a:138).

DEVELOPMENTAL SEQUENCES

Three developmental levels

In Piaget's (1970) model, knowledge arises through a subject's (agent's) actions on objects, i.e. an *S-O* relation. The basic point running through the several formulations in Box 1 is the construction of objects. Three levels in the sequence correspond to infancy, childhood and adolescence. [6] One level occurs during infancy as practical intelligence. Its completion is the construction of actual objects, i.e objects in the actual world. A

second is representational thought which occurs during childhood as the construction of abstract objects applied to contexts in the actual world. This level sub-divides as preparation (preoperational) and completion (concrete operations, i.e. operations on actual objects). The third is formal understanding, occurring during adolescence as the construction of abstract objects whose contextualisation is not restricted to the actual world which is merely one part of reality. This level too has a comparable sub-division of early and mature forms of formal operations.

(Box 1 about here)

Piaget used the terms *level, stage* and *period* inter-changeably. Equally variable was the number of levels – sometimes three (1970), sometimes five (1950b). This inconsistency is more apparent than real. An analogy may help by regarding a developmental level as a contour line on a map. Contours chart the shape, slope and height of a mountain; their ordering is exact; fine-grained contours map onto large-scale contours; contours are not invalidated in virtue of their insensitivity to other properties of mountains (geology, landscape, climate). This analogy has a weakness. It breaks down in that mountains have a determinate height in contrast to the development of knowledge which is open (non-determinable). It also has two specific strengths. One is that maps do not determine routes on mountains, but they do chart the levels of any route. In the same way, a model of complex learning (equilibration) does not determine developmental pathways. The other strength is that contours are properties of mountains, not mountaineers. Reinhold Messner has climbed all fourteen 8000m Himalayan peaks, but this feat does not preclude him from taking a stroll at sea-level through the streets of Venice. In a similar way, developmental levels are levels of knowledge, not level of knowers. Infant-level behaviour in an adult is not a contradiction in Piaget's model.

Three developmental levels – but levels of what? There is an answer in epistemology. Epistemology is the theory of knowledge. A central problem is "What exactly is knowledge?" This problem is normative and non-empirical. And it is still under dispute. Even so, one condition of knowledge has never been challenged in that knowledge entails the truth of what is known (Moser, 1995). No truth means no knowledge, and so no development of knowledge. Piaget realised that this normative problem had an empirical counter-part "How does knowledge develop?". This amounts to a productive problem-shift, the conversion of a normative into an empirical problem. This problem is central to Piaget's (1950a) developmental epistemology, one of whose questions is how true knowledge develops during childhood (1923:57; 1995a:184). So the short answer is: three levels of knowing in the development of knowledge

<u>Formation of knowledge</u>

In this model, knowing is a process whose outcome is the formation of knowledge. Two processes were identified by Piaget (1967: 6), access and constitution. Access is a causal process to be investigated through causal facts. This type of investigation is well known in psychology. Constitution is a normative process directed on coherence, closure, and necessity, normative because these are normative properties. The series of natural numbers is coherent: each number is unique and compatible with the others. It is closed: operations on numbers generate numbers. Numerical truths are all necessities: 7 + 5 = 12

is, and has to be, true. In Piaget's model, the normative constitution of knowledge was investigated by reference to normative facts. This distinction is revisited in section [4]. Right now, it is introduced through examples in Box 2. Notice that both causal and normative facts are empirical. Access and constitution are concurrent, leading to partial but progressive advance over time. Isaacs (1951) aptly summarised Piaget's model as a 'psychology of normative facts'.

(Box 2 about here)

Access alone is not enough. First, actions have both factual and normative properties. '*Action necessarily deforms the ideal in virtue of its mixture of fact and norm'* (1918:116). An action can be causally successful and normatively unsound.

- Presented with box-shaped figure whose five visible sides are white, one child argued that the back was also white *because the box is all white and so the back can't be another colour*. This response is successful in that this child has given an answer to a question in this causal context. Yet there are three (normative) fallacies in this child's reasoning. (i) Is there a sixth side at all? (ii) If there is, is it white? (iii) If it is, does it have to be white? (Piaget, 1987a: 31).

Action is based on an assimilation which is the capacity to search for knowledge, but this 'search for coherence' never guarantees success (1985: 6, 13). Error (false belief) is always a possible outcome, not merely true knowledge. Indeed, a central developmental problem is the advance from causal fact to logical norm (1995a: 51). This leads to the second reason. Knowledge falls into systems with internal properties, such as consistency, closure and necessity (1986: 302-3). Thus knowledge requires organisation. Otherwise questionable reasoning can be manifest. Mental organisation may well change over time in any of three ways. And this can be a change resulting in a better organisation.

From level 1 to level 3

There are reckoned to be two major advances. One is from success to understanding. Infant activity is confined to 'success or practical adaptation, whereas the function of verbal or conceptual thought is to know and state truths' (1954: 360; 1953a: 240). An activity is successful if the agent's goals are met. But goals are intentional in virtue of the beliefs and desires, intentions and values within the agent's 'horizon of intentionality' (von Wright, 1983). This "horizon" is augmented, in Piaget's model, on the basis of the functioning of its own elements, resulting in "better" levels of organisation within any "horizon". The second advance is from representational to formal understanding. This requires actions to be directed less on particular contexts and more on systems with infinite implications. This advance runs from the 'causal to the logical' (1995a:51). It is generative of acts of judgment. 'To judge is....to assimilate; that is to say, to incorporate new content into a prior structure in an antecedently elaborated systems of implications (1953a: 410*). An act of judgment is the is the acknowledgement of the truth of what is known (Smith, 1999b, c). Any act has causes for psychological investigation. Any judgment is based on reasons for epistemological investigation. To develop intellectually is to improve normative control over judgments made under causal conditions. In general, 'all knowledge can be considered as being relative to a given previous state of lesser knowledge and also as being capable of constituting just such a previous state in relation to some more advanced knowledge' (1950a:13). The proposal is that all knowledge has bi-directional relations with linkages to both origins and successors. Knowledge never

emerges *ex nihilo*, from nothing (Inhelder & Piaget, 1964:285). Nor does knowledge converge on a limit, *a priori ad quem* (1972).

CRITERIA OF DEVELOPMENTAL LEVELS OR STAGES

Piaget (1960) stated five stage criteria of his three developmental levels. These criteria deserve individual comment with the proviso that they are jointly – not severally – applicable (see Box 1). A pseudo-criterion should also be noticed first.

A pseudo-criterion

Chronological age is *not* a criterion. Agreed: age references were included in Piaget's empirical reports. Variability in age was specifically covered by criterion 1. But ages were indicators, not criteria. Men who wear dresses have not on that account changed their sex, even though wearing a dress is an indicator of the female sex. Similarly, evidence that *preschoolers* can operate at one level, when Piaget's evidence was based on youngsters in *mid-childhood*, misses the point. A (normative) criterion and an (empirical) indicator are not the same thing (Smith, 1993, sect.18). An indicator has relative utility: it is more or less useful in sample selection. A criterion has dichotomous validity: anything that satisfies criterion of *C* is, and has to be, a *C*; anything that does not satisfy it is not, and could not be, a *C*.

Constancy in Order of Levels

One criterion is constancy in order of levels. This criterion is reliable (Müller et al., 1998, 1999; Shayer et al., 1988). Ordering can itself be interpreted as novel construction or 'the creation of new objects' (Inhelder & Piaget, 1980: 21*). There are two main classes of objects. One comprises *actual objects*, familiar objects in the actual world – such as tables, clocks, and people - with their spatio-temporal and all manner of other properties. But these are not the only objects. Reality comprises *abstract objects* as well, such as number or society in their infinite complexity in virtue of subtle properties.

- construction of actual objects. Object permanence (conservation) is a prerequisite of further advance to objectivity. Experience without this would be 'less even than a dream'(Kant, 1933). This is why Piaget demarcated tableaux and objects during infancy (Smith, 1998). This is no easy matter: young infants do not search for unobserved objects at all, or search selectively, making the A-not-B (stage IV) error (Piaget, 1953a, 1954; cf. Moore & Meltzoff, 2000).
- construction of properties of actual objects. Properties are abstract objects. A pack of cards is one pack and fifty-two cards, so these incompatible properties (1, 52) are not reducible to one actual object (Smith, 1999a). Number conservation is a paradigm case because the non-conservation of number is blind to this incompatibility (Piaget, 1952a; cf. Bryant, 1997).
- construction of systems of properties. These require conservation and novelty, conservation as a secure grasp on the past and novelty as a bridge-head to advance. Proportionality is a paradigm case whereby a new relation is imposed on available (conserved) relations (Inhelder & Piaget, 1958; cf. Bond, 1997). Analogical reasoning, which is non-quantitative, is a precursor during childhood (Piaget, 1986, 2000; cf. Goswami, 1992).

Integration of Prior Levels of Knowing Conserved

Integration amounts to convergence through advance, i.e. nothing is taken away even though something is added. Thus formal operations require concrete operations. This dependency should be respected in task-design. It has been argued that this criterion has been disregarded in the selection task applied to formal operations (Bond, 2001; Adey & Shayer, 1994; Smith, 1993: 117).

Bi-Directionalty: Current Knowing From Previous Knowing To Novel Knowing

Preparation is the principal feature of preoperational thought whose successful completion is concrete operations, which in turn are preparatory to early formal operations. The implication is that task design should have a bi-directional focus both on what preceded and what could succeed any epistemic action. Inhelder's (2001) tasks are exemplary in this regard, covering both phases of both concrete and formal operations.

Formal Model

Piaget (1953b) denied that all human thought can be formalised. But he also regarded logic as the formal science of truth (Piaget, 1949:4; 1971:35). [7] Piaget formalised his structures in three ways, one by means of group theory (Piaget, 1949, 1950b), then through category theory (Piaget, 1992; Piaget, 1977), and finally through entailment logic (Piaget & Garcia, 1991). There is an outstanding question here about operationalisation. Developmental order (criterion 1) is well defined through these formalisations. Developmental research has tended to focus on Piagetian tasks as the basis of their evaluations. How, then, is formalisation linked to tasks? The answer is not clear. This means that methodological gains in experimental design may have a theoretical cost, amounting neither to a verification nor falsification of constructivism (Vonèche, 2001). And this leads to criterion [5]

Equilibration

The final criterion is equilibration. So much is implied by Piaget's *tertium quid*. If there are developmental levels, some mechanism of advance is required. Equilibration purports to be such a mechanism. Commentators have repeatedly interpreted research evidence as a compelling refutation of Piaget's stage-theory (Case, 1999; Flavell et al., 1993). In its own terms, this interpretation is compelling. The trouble is that "its terms" have two massive limitations. One is a false-negative: equilibration is typically denied a place in these interpretations of the evidence. This means that a *tertium quid* commitment has been given up. The other is a false-positive: alternative interpretations are on offer as interactions between the "two lines". This results in the same non-commitment.

In short, three developmental levels defined by five criteria through two main advances in the development of knowledge. The outstanding problem concerns equilibration as a developmental mechanism.

DEVELOPMENTAL MECHANISMS

Equilibration as a developmental mechanism – well then, three questions arise:

- Is there a testable model of equilibration? Bryant (2001) has elegantly reckoned

that there isn't a discernible way to test Piaget's model, and this view has long been standard (Flavell, 1963:244).

- Is there a formal (theoretical) model of equilibration? This has been insightfully denied by Klahr (1999) in what is, once again, the standard view (Boden, 1979).
- Does the absence of a (testable, formal) model mean that equilibration is thereby a construct well past its "best by" date? The answer is the same – "No".

This last answer has several lines of independent support:

Equilibration: neither a testable nor a formal but an intelligible model

(i) Piaget's construct has been persistently misrepresented in developmental psychology and elsewhere (Lourenço & Machado, 1996). Thus the demarcation between what is being rejected and what is being missed has been eroded.
(ii) Plato long ago stated a problem which is the construction of (atemporal) necessary knowledge through (temporal) equilibration, and it is this problem which is central to Piaget's research-programme the implications of which have not been systematically worked through in developmental theory and research (Smith, 1999d).
(iii) A complementary problem arose in Inhelder's (2001) work as the construction of (temporal) procedures from atemporal structures of action. This is productive problem-shift which continues to be under-exploited (Vonèche, 2001).
(iv) Piaget's construct has a family-resemblance to models of systemic organisation in the life sciences, which in any case should not be understood on the basis of non-organismic models in the physical sciences (Chapman, 1992).
(v) Piaget's construct has the potential to be assigned a causal interpretation in terms of self-organising processes (Molenaar & Raiijmakers, 2000). The model matches the criteria of adequacy of equations which underpin explanatory theories in science.
(vi) A central problem in the life sciences is the problem of emergence. This problem has to be confronted by any adequate model of development. Key aspects of Piaget's construct are invoked in interactivist perspective applied to emergence (Bickhard, 2002).
(vii) All interactions are value-laden, which leads to the problem of normative exchange which is assigned a distinctive interpretation in Piaget's construct (Mays, 2000).
(viii) Piaget's construct straddles both causal and normative domains (Brown, 1996, 2001). This is a strong point both in itself and in virtue of the total absence of any alternative model which combines both.
(ix) Comparable problems occur in both individual and social interactions (Müller & Carpendale, 2000) and Piaget's construct has the potential to present a unitary account of both.
(x) The link between developmental change and the nature of knowledge was central to Piaget's corpus of research and theory, and its systematic reevaluation has yet to be done (Beilin & Fireman, 2000).

In short, despite two major deficiencies, Piaget's construct is stated to have a degree of intelligibility and fitness of purpose. What now follows are some principles which could serve in a better representation of this construct. Five principles from Piaget's (1918) model are shown in Box 3. Five epistemological principles which fit Piaget's work are show in Box 4. Both sets are now reviewed.

FIVE PRINCIPLES IN PIAGET'S MODEL

Five early principles are shown in Box 3. Each is now reviewed through later work.

(Box 3 about here)

Action, organisation and identity.

Assimilation is a property of action. This notion was specifically invoked with its complementary, initially regarded as imitation (1928), and later changed to accommodation (1953a). What remained the same was a functional role in the intelligent search for knowledge. This search is a search for coherence in the conferral of meaning on the objects encountered in action (1985:13,16). This search results in degrees of organisation, present to some degree at all levels of development (2000). What changes is the extent of this logic. Any level of organisation amounts to "one" construction (assimilation) of the world. Any construction has its own identity: it is what it is at the level of that person's point of view. But is inherently vulnerable to qualification or even disqualification in virtue of the complementary and concurrent, process which is accommodation to new content. The problem is how to fit "new" content to "one" available structure. As Piaget and Voyat (1968:2) presciently remarked, 'out of all the logical principles, the principle of identity is perhaps the one which remains the least self-identical throughout its development'. Actions are consecutive throughout a person's life, such that each new action can preserve the organisation of a predecessor or can replace it, wholly or in part.

Action as the basis of knowledge.

This principle places the origin of knowledge in action on objects (1970:104;1971:6-8; 1995a:70-71). There are two points here. First, practical intelligence is based on infants' activities which is the basis of all further development. Actions are actions on objects, and two main classes of (actual, abstract) objects were identified under Piaget's stage-criterion [1]. Second, representational and formal thought are based on actions or operations on abstract objects contextualised on actual objects initially and then contextualised independently at higher levels. These operations are what a person knows how to do in relation to these objects. In this model, what matters is not what (knowledge, concepts) anyone has but rather what is done with them. This point is captured in Piaget's claims about the regulation of action: 'we speak of regulation when the results of an action, *A*, modify the repetition of that action *A*'. Regulation can therefore take the form either of correcting *A* (negative feedback) or of reinforcing it (positive feedback)' (1985:16). The requirement for the presence of at least two actions has the consequences for the extent of the "gap" between them. This "gap" was officially recognised by Piaget (1941:268-70) as horizontal (within-level) *décalage* which occurs in four forms: identity, vicariance, correspondence, analogy. Vertical (between-level) *décalage* occurs through the three levels in Box 1.

Piaget's position fits Goethe's famous dictum *Im Anfang war die Tat*. [8] This dictum rules out a basis of knowledge in language. Piaget's position can be clarified through a parallel with Frege, both of whom had the same view of logic as the formal science of truth (Smith, 1999b). As the architect of modern logic, Frege's (1972:106) aimed 'to break

the power of the word over the human mind'. Piaget was likely to have learned about Frege's work whilst still at school and had a similarly cautious view about language (Smith, 1999a). But Frege had a realist view of the abstract objects of logic in contrast to Piaget's constructivism which located logic in the general coordination of actions. 'Each new action exactly by realising one of the possibilities generated by previous actions itself opens up a whole web of not previously conceivable possibilities. It is there, in the relation of causal reality with the possibilities opened by it where these possibilities are tied together by a link of virtuality which is ever nearer to logical implication, that is to be found the solution to the central problem of atemporal norms and actual development' (1950a:34). In this model, actions have causal and normative properties and each new action has the capacity to link back to the structure of previous actions or to generate new possibilities for better levels of organisation in the structuration of the world.

Action as normative fact

Any action combines causal and normative properties in that although '2 does not "make" 4, its meaning "implies" that 2 + 2 = 4' (1971:49). A model of causal facts could not explain this distinction. Causality (making something happen) is not entailment (implication). A abstract number such as 2 could not be part of a causal interaction on actual objects. An arithmetical truth could not have its basis in the causality of human action which is just as capable of leading to error as to truth. It is for these reasons that normative facts are in the reckoning. Normative facts are:

> facts in experience permitting the observation that subject such-and-such considers him- or herself to be obligated by a norm, irrespective of its validity from the observer's point of view (1950a:30);
>
> imperative rules whose origin is in social interactions of all kinds, and which act causally, in their turn, in the context of individual interactions (1995a: 69).

There are three claims here with a fourth to be added. One is that normative facts are modal with an obligatory or necessary character. Second, normative facts are intersubjective by virtue of their dual instantiation in both social and individual contexts. Third, development occurs as the reciprocal interplay between access to and the constitution of knowledge, between empirical opening and normative closure. [9] Fourth, normative facts themselves evolve during ontogenesis as too in the history of science (1995a:166).

A model of normative facts requires a model of causal facts since the knowledge gained by humans is in fact always mediated by actions which always have a causal component. But this is not enough, necessary but not sufficient. Knowledge has a normative component bound by relations of meaning and entailment. This difference is fundamental in developmental epistemology.

> A norm is an obligation, and no obligation can be derived from an observation. That is clear. Even so, whilst consciousness embodies.....or applies norms and therefore is not speaking in the language of facts but rather of *normative truth*, the developmentalist who is confined to facts of experience open to control for each observes, without taking sides for or against, *that norm's influence on the consciousness which instantiates it.* From this point of view, any norm is also a fact (1950a:29-30; my emphasis).

A fact (what is the case) never entails a norm or value (what ought to or should be the case). This is "Hume's rule" (von Wright, 1983). This distinction was accepted by Piaget, whose study of normative facts does not breach "Hume's rule" just because normative facts are facts (1995a:124). Such facts have a constitutive property in that a norm is internal to a normative fact (eg: a 1-metre ruler made of wood). Third, it is an open question whether an agent recognises the norm which is implicit in any action. This means that the investigation of normative facts centres on how a specific norm is in fact used by a specific knower. Is it ignored altogether (Freudian denial)? Is it converted into a (Vygotskian) pseudo-concept? Is it a fully recognised (Piagetian) operation? There are many levels here, as attested by these three sages.

This distinction runs into two objections. The first objection is that the causal-normative distinction is irrelevant to psychology. In psychology, Piaget's model gets half-marks since it deals with facts – but stick to causal facts and leaves the norms out. The counter-objection is that this misses the point. First, the disjunction "causes or norms" does *not* turn on whether causes are empirical, and norms not so. Causal *facts* and normative *facts* are both empirical. Second, the explanation of causal facts is sufficient in psychology directed on the study of *people* in causal settings. But psychology alone is not enough with regard to the development of *knowledge*. Causal facts are insufficient here because knowledge has normative properties. If these properties are left out, there is no way to secure fundamental distinctions. Here are two examples in which (normative) distinctions are lost in causal studies exclusively directed on "correct responses".

- *epistemic fallacy: belief does not entail knowledge*.

Although falsehoods can be believed, only truths can be known (Sosa, 1995). It follows that mere belief never amounts to knowledge: the belief might be false. Nor is true belief the same as knowledge, otherwise the distinction between belief and knowledge would collapse. How is this fallacy avoided in a causal study directed on "correct responses" which may amount to true belief without knowledge?

- *modal fallacy; truth does not entail necessity*.

True knowledge and necessary knowledge are not the same thing (von Wright, 1983). Any (empirical) truth could have been otherwise; yet any necessity could not be other than it is. It follows that true knowledge never amounts to necessary knowledge, unless some further normative condition is met. How is this fallacy avoided in a causal study directed on "correct responses" which may amount to true knowledge without necessity?

The second objection concerns the relative priority of causal and normative facts. In Piaget's empirical studies, causal facts are introduced in task design through instantiation. But instantiation is not systematic investigation. In consequence, causal facts were assigned too low a status in relation to normative facts. There are two replies to this objection. One is to agree with it. Indeed, Piaget (1952a:149) said as much himself, declaring a lack of interest in the experimental manipulation of causal facts. But this concession to the effectiveness of the empirical critique of Piaget's work is severely limited. An approach which is 'exclusively psychological and insufficiently epistemological' (Piaget, 1963) won't do. Yet such has been the pre-occupation in psychology (causal facts) without enough regard for developmental epistemology (normative facts). The implication is clear: a dual pre-occupation is required. Otherwise,

this second objection reduces to the first, already rejected objection.

Normative facts as disequilibrium or equilibrium.
Piaget (1980:xv) identified three varieties of disequilibrium: (a) instability if the same action is coupled with different results; (b) non-compensation if contrary actions are not self-cancelling, and so generate an intermediary; (c) incomplete inference if action-coordination precludes necessity. These three varieties can be exemplified in number conservation (1952a, pp.43-45). Asked to place just enough glasses to match six bottles in line on a table, Bon laid out 12 glasses from the nearby tray, i.e. the same action (laying a table) has different results ($6 \neq 12$). Gal succeeded in placing six glasses next to the six bottles, but then declared that there were more bottles, when the glasses were bunched together, and more glasses, when the bottles were bunched together, i.e. two lines are attributed both equal and unequal numerical properties ($6 < 6 = 6 > 6$). Mül counted correctly the six bottles and six glasses, but then denied this number-identity, i.e. the denial precludes the necessity of the numerical identity ($6 = 6 \neq 6$). Disequilibrium does not amount to contradiction within a train of thought, but occurs in the self-regulation of action (Piaget, 1980:xiv). This is a principal reason why access to a higher level of equlibrium is required to overcome disequilibrium at a lower level. It follows that metacogniton or hypercognition (Demetriou, 1998) is an integral feature of Piaget's construct.

Processes of equilibration.
Due to the lack of a testable or formal model, the temptation to interpret equilibration by analogy is almost irresistible. Piaget invoked mathematical and physical analogies, and also noticed their limitations. Thus his distinction between *ideal equilibrium*, which is not in fact attained, and *real disequilibria*, which are all that is left to humans in causal settings, was sometimes interpreted by reference to the notion of a mathematical limit, i.e. truth (ideal equilibrium) is a limit towards which any actual disequilibrium converges. Although Piaget (1918:46; cf. Chapman, 1988) did use a mathematical analogy, it simply postpones the epistemological problem. Convergence on a limit is lawful in mathematics, but is problematic in this model just because the analogy begs the question about whether and to what extent equilibration is lawful. Analogies based on the physical sciences break down, and were officially stated by Piaget (1985: 61) to be misleading. A physical explanation of the transformation in a glacier from ice-to-water through an equilibrial line is inconclusive when applied to the development of knowledge. Water is not a development of ice; yet *savoir* (knowing that) is reckoned to develop from *savoir-faire* (knowing how) in Piaget's model. Even so, analogies can clarify – though not clinch - a point. Thus the analogy used in [2] served to clarify developmental levels as contour lines contour lines.

OPERATIONALISATIONS OF EQUILIBRATION

There have been two operationalisations of equilibration. One was based on probabilistic processes (1968). This model is patently defective since implication is logically necessary, and so is not the same thing as stochastic likelihood. It follows that this model could not explain the advance from the causal to the logical. Piaget (1985) later offered a second account. [10] This account raises questions. First, is it consistent? The account is a

retrospective account used by Piaget in the re-analysis of his own studies. Its main conclusion is that however diverse the ends of action and thought, 'the subject seeks to avoid incoherence and for that reason always tends toward certain forms of equilibrium. (This) is never achieved except in terms of provisional levels....(such that) in every area disequilibrium plays a functional role of first importance because it necessitates re-equilibrations' (1985: 139). This conclusion is congruent with the principles set out sixty years earlier by Piaget (1918). But its systematic evaluation is required. The advice given earlier by Beilin and Fireman (2000) is well taken, but beyond the scope of this paper. Second, is the later account complete? Here are two reasons why it isn't. One is that a model of value exchange was also set out by Piaget (1995a). This account too requires interpretation and evaluation in the context of the 1975 account (Müller & Carpendale, 2000). Another is that it has been complemented by another in Piaget and Garcia's (1989) intra-inter-trans triad as a distinct process working inter-dependently with equilibration. This is an effective contribution to the proliferation of interpretations! What is required is a comprehensive and critical analysis of these models, taken together. This is beyond the scope of this paper, which now takes an epistemological detour.

Five epistemological principles: AEIOU mnemonic

The detour is Piaget's critique of Kant's epistemology. Kant (1933: B20) gave a famously affirmative answer to the question "*How is knowledge possible*?" Notice that this question is normative. Piaget sided with Kant by regarding human knowledge as objective. A comparable position is assumed in recent perspectives on constructivism (Chandler, 1997). But Piaget (1950a:12) also criticised Kant in view of the incomplete answer given to Piaget's counter-part question "*How does knowledge develop*?". Notice that this question is empirical, and it points to psychology which is the empirical science of the human mind. This counter-part question is central to Piaget's developmental epistemology. Piaget (1950a) realised that its answer required psychology to be augmented. Causal facts would be in the reckoning, but along with "other facts" for empirical investigation. These are normative facts. This amounts to a productive problem-shift. What Piaget wanted to know is how normative facts develop during children's develop in line with his 'research-programme' (1918:148). Five normative principles (see Box 4) are now reviewed followed by an example.

(Box 4 about here)

Autonomy Self-indulgent thinking and autonomy are not the same thing (1998:165, 213, 259). Rather, autonomy is the reconciliation which is required when 'the ego's interests conflict with the norms of truth' (1998: 124), the self with normative rule (1995: 241). Each individual should be a free agent without free license. This is a commitment not to the individualism of the solitary knower, but to the *invidualisation of knowledge* whereby 'each individual is led to think and re-think the system of collective notions" (1995a:76). Teaching is a case in point. Piaget (1995a, 1995b, 1998) accepted that teaching can be efficacious in new learning, an admission denied by Bryant (1995). Equally, teaching can amount to 'normative pressure' (von Wright, 1983), which results in heteronomous thinking ("this is how we do it, so this how you ought to do it"). At issue in Piaget's model is how the latter is converted into autonomous (spontaneous) thought.

Entailment/Equality Necessity is constituted through relations of entailment and equality (Marcus, 1993). They are central to necessary knowledge. Since necessity along with possibility are modal concepts, they are central to modal understanding. A paradigm example is the development of necessary knowledge. Evidently, Piaget stated this to be central to his work for half a century (Smith, 1999d). [11] This is not a surprise. Necessary knowledge continues to be central to epistemology. It is one thing to use an arithmetical rule correctly; it is something else again to recognise that an answer 'had to be so, and could not be otherwise' (Smith, 1993: 63).

Intersubjectivity The concept of intersubjectivity is variously used in reference to social interaction or to shared culture. Neither means the same as self-identical thought. Representational thinking is subjective – to each his or her own. Grasping the thought which is the Pythagorean theorem is intersubjective. This is because two thinkers can have in mind "the same thing". This common ground is the intersubjectivity of thought. How else could different acts of representation be about "the same thing" (Smith, 1999b, c)? At issue in Piaget's model is the conversion of representational thinking into intersubjective thought. In Piaget's model, the use of logical structures secured intersubjective conversion in virtue of 'a general logic, both collective and individual, which characterizes the form of equilibrium common to both social and individual actions' (1995a: 94).

Objectivity Psychology is indifferent to the fundamental distinction between truth and falsity. This is because there can be a good psychological explanation of human error (perceptual illusion, "false" memory). It follows that not every property of thought can be explained by psychology (Smith, 1999b). Truth is one such property. It is for this reason that Piaget (1928: 239) insisted on a clear demarcation between logical and psychological properties. This led to the problem of the origin of "true knowledge" in the human mind (1995a: 184) through acts of judgment directed on the recognition of the truth of what is known (1953a: 240).

Universality The term *universality* can be understood in two different ways in psychology, one about the transfer of knowledge, the other about the universalizability of knowledge. This former is a standard problem in psychology about individual, contextual, and cultural differences in knowledge-domains. Thus some children do not transfer their school knowledge of arithmetic in applied contexts, whilst other children do not transfer their street-arithmetic to the classroom (Bryant, 1995). Successful transfer depends on multiple achievements. What Piaget (1918:46) wanted to know was something else, namely how anyone can universalise their knowledge, given that its origin is always in particular domains, contexts and cultures. How does the universalisation of knowledge occur from "in this context, all the daisies are flowers" to "all daisies are flowers", from "in our culture, 6 = 6" to "it is always the case that 6=6" (2000: ch.6)?

Case Study: children's reasoning by mathematical induction

The AEIOU quintet can be illustrated through an empirical study of children's reasoning by mathematical induction. The study is reviewed in Box 5.

(Box 5 about here)

An extract from John's reasoning is instructive. John (aged seven years) had added an equal number of plastic counters to two containers, one containing one at the outset (*A*), and the other empty (*B*). The interview included repeated questioning about serial addition to *A* and *B*, including:

Interviewer	How about if you put a great number in that one and a great number in that one. Would there be the same in each or more there or more there?
John	That would be right up to the cover in the sky and that would be right up to God, so then they would still have to be more

This is a superb example of analogical reasoning. After confessing my ignorance, John explained his analogy. God is in Heaven and the number in *B* is so big that it reaches God. But the number in *A* is bigger, so it would go up to the cover over Heaven. This reasoning also has five epistemological properties:

- autonomous John's reasoning was his own. It was a free mental act which was "spontaneous thought". This is the highest type of response in Piaget's (1929) categorisation of responses in contrast to "provoked" responses in receptive understanding (1952a).
- entailment John's conclusion concerned "what has to be". All mathematical truths are necessities, as too are all logical deductions (Smith, 1997a).
- intersubjective John's reasoning was a valid argument. This amounts to *his version* of Euclid's axiom (*if equals are added to unequals, the wholes are unequal*). This axiom is not in the Mathematics National Curriculum at KS1 (DfEE, 2000).
- objective John's answer was correct. The difference in correct responses between the children in School Year 1 and 2 was significant ($\chi^2 = 28.657$, df. (1), $p < .000$).
- universal Over the whole interview, John correctly answered two criterial questions relevant to mathematical induction. This amounted to the universalisation of knowledge.

This example secures four points about Piaget's (1918) research-programme. One concerns its ontology, or the intrinsic properties of the phenomena to be investigated. This includes normative facts, manifest as the use made by particular individuals of general principles, such as the AEIOU epistemological quintet. A second concerns its methodology. The method used here was Piaget's critical method (Smith, 1993:58). The use of normative principles has to be shown – by the user, not just the investigator in task design. A third concerns development. What requires systematic investigation is development in the use of these epistemological norms, and others like them. This huge under-taking has still to be carried through. The fourth concerns equilibration. A model which is exclusively causal or exclusively normative could not in principle account for the development of knowledge. Equilibration is a construct which combines both. Although this construct is currently neither testable nor formal, it has an intelligible function at work "between the two lines" of heredity and culture. The mechanism is actual reasoning with the potential to recast past reasoning in a present context for future re-construction.

IMPLICATIONS

My argument has three implications, one about causal and normative facts, another about acts of judgment, and a third about empirical methods.

Causal facts and normative facts

One implication concerns causal facts and normative facts. Under this interpretation, a model of the development of knowledge requires a joint focus on both causal facts and normative facts. This does not rule out the exclusive study of causal facts in human development. But it does rule out the exclusive evaluation of a joint model through such a study. Knowledge has normative properties to which psychology with a focus on causal properties alone is officially blind. A valid psychology of "false" memory is spectacularly insufficient in an account of true knowledge. The normative facts in Piaget's model were epistemological. Equilibration as its central construct was envisaged to be both causal and normative. Crucially, a study with a joint focus would contribute to an empirical account. There are three ways in which future study could move ahead. One is to re-visit the (epistemological) norms used by Piaget with parity of focus on both causal and normative aspects. This route would avoid two deviations, the over-valuation of normative facts and the over-valuation of causal facts. Another is to augment the present stock with other epistemological norms. A notable case in point is a deontic logic of action (von Wright, 1983). A third is to generalise the axiological framework by reference to other values in all domains, and not merely scientific values (Brown, 1996, 2001). [12] The importance of this general problem continues to be acknowledged: 'how is it possible that creatures like ourselves, supplied with the *contingent capacities* of a biological species whose very existence appears to be radically accidental, should have access to *universally valid methods of objective thought*?' (Nagel, 1997: 4; my emphasis).

Acts of judgment

A second implication concerns acts of judgment in which something is recognised as true, realised to be true, acknowledged that it is true. Any such act has inclusive properties which are both causal and normative. An *act* (action, operation) is performed in causal settings and mediation through action is the basis of all knowledge gained by humans. A *judgment* has normative and meaningful implications which are central to the development of all knowledge. There is a difference between evidence about:

- an act displayed on a task designed to be compatible with a particular norm
- a judgment due to the use by an individual of a particular norm

An individual may be one person, a group of collaborating individuals, or a social institution. There are all the intermediaries between a (correct) response in line with truth and a judgment in which some reason is given for the truth of any response. This point generalises to other normative principles in the AEIOU quintet and elsewhere. The only way in which this distinction can be drawn is by reasoning with due attention to reasons which amount to good reason (Smith, 1993:sect.13; 1997a:233; 1999d:25). Such reasoning would include some combination of conservation and novelty, the best of the used in the present for the development of future knowledge. The empirical question of how normative control due to any agent changes through actions over time is an open question.

Empirical methods

A third implication concerns empirical methods. An experimental method is used to generalise causal regularities from a sample to a population. A critical method is used to demonstrate that a (general) reasoning pattern is instantiated in a sample. These are distinct methods with distinct functions. Central to a critical method is an ontology (phenomena, problems) in which reasoning is on display as used by the reasoner, and not merely by the investigator. Evidence from a critical method could be useful on three fronts: (a) the respects in which children understand one and the same principle on a task standardised by an adult; (b) how well or badly reasons fit responses in human reasoning; and (c) the range of novel thinking (cf. John's superb analogy about the cover over heaven). A further feature of this method is its convergence with the standard analysis of knowledge in terms of true belief which is justified by the knower (Smith, 1999e).

In short, my argument is that the Grand Tower project in [1] can be continued through these three implications.

Footnotes

1. See also autobiography (Piaget, 1952b, 1972); biography (Barrelet & Perret-Clermont, 1996; Smith, 1997b; Vidal, 1994); websites: *www.piaget.org* and *www.unige.ch/piaget*

2. Unless otherwise indicated, all references are to Piaget's work.

3. Piaget (1970) disowned the attribution "Piaget's theory".

4. * indicates my amended translation.

5. Inter-dependently with, not independently of. Equilibration operates "between the two lines".

6. This common-sense distinction is captured in Piaget's "stage theory".

7. This is the standard view of logic (Frege, 1979:128; von Wright, 1983:130).

8. *In the beginning was the deed,* whose contrary in Goethe's *Faust* was *In the beginning was the word* (John 1-1).

9. *ouvertures et fermetures* (in Smith, 1993:171).

10. The earlier translation (Piaget, 1978) is radically defective and has been re-translated (Piaget, 1985).

11. Examples covering fifty years are in Smith (1999d). For example: 'the emergence of logical necessity constitutes the central problem in the psychogenesis of logical structures' (Piaget, 1967:391);

12. The irony here is that the starting-point in *Recherche* was the clash between science and faith, how objectivity in science and truth due to faith can be reconciled (1918:21).

Box 1 Criteria of developmental periods or stages

- Criteria of developmental stages or periods

1. a distinct chronology in the sense of a constant order of succession. The average age for the appearance of a stage may vary greatly from one physical or social environment to another
2. the equivalent of an integration in the transition from a lower stage to a higher one
3. in any stage, it should be possible to distinguish an aspect of achievement with respect to the stages going before and also an aspect of preparation with respect to the stages coming after. Naturally it is possible for both achievement and preparation to be promoted or hampered by favourable or unfavourable external situations
4. all the preparations leading to a stage and all the achievements characterising this stage (are due) to the existence of a general (or total) structure
5. (successive stages form) a series of equilibrium levels, the fields of which would be always more and more extensive and the mobility always greater, but whose increasing stability would depend precisely on the degree of integration and of structuration

Source: Piaget (1960, pp.13-14)

- Three periods of mental development

Practical Intelligence: indicative ages: preparation 0 – 7 to 9 months; achievement 9-18 months
Representational Thought: indicative ages: preparation 1_/2years – 7/ 8 years; achievement 8 – 11 year
Formal Understanding: indicative ages: preparation 11 to 13 years; achievement 15-20 years

Source: Piaget (1970: 110); Piaget (1972)

- Three periods of social development

'In individual mental development, which is a progressive equilibration and therefore does not involve the essential duality between diachronic and synchronic factors, the transition from causality to implication involves three basic steps having distinct proportions of these two forms of relationship: rhythms, regulations and groupings.'

Social rules, values and signs are meaningfully interpreted (assimilated) as
rhythms: social activities and routines
regulations: social interaction and practices
groupings: social structures

Source: Piaget (1995a, pp. 45, 56)

- Three periods in the development of modal knowledge of necessity and possibility

'The first is one of nondifferentiation: reality includes many pseudonecessities, whereas possibility consists in simple, direct extensions of actual realities.

The second period (coinciding with the formation of *groupings* and *concrete* operations) is one of differentiation of the three modalities: possibilities unfold into families of co-possibilities; necessity transcends the local coordinations, generating operational compositions determining the necessary forms; and reality consists in concrete contents.

The third period, finally, is one of integration of the three modalities within a total system so that reality appears to the subject as a set of actualizations among others that are possible. But it is simultaneously subordinated to systems of necessary connections.'

Source: Piaget (1987b, pp.4-5)

Box 2 Causal Facts and Normative Facts

- Reasoning about relations

Causal facts. Children use language in their cognitive performance. Is their linguistic use of *bigger/smaller* a cause of cognitive performance in placing a new stick in a series of big-small sticks? Or is it the other way round? In his review of training studies of seriation, Bryant (2001) argued for the linguistic hypothesis as an interpretation of the evidence.

Normative facts. Children's social knowledge of family relationships change with age. What also changes is their understanding of the (normative) aspects of these relations. If Alan and George are brothers whose sister is Nicola, the relation brother of has three properties: (i) *irreflexive*: Alan can't be his own brother; *symmetrical*: if George is Alan's brother, Alan is George's brother; *transitive*: if Nicola is George's sister and George is Alan's brother, Nicola is Alan's sister. Alan knows the names of his brother and sister, implicitly accepting (i). But his qualifications over (ii) and (iii) verge on denials (Piaget, 1992:97*):

Does George have a brother?	No
Are you George's brother?	A bit. not really
What is necessary to be someone's brother?	You have to love him
Does your sister have a brother?	Yes, George
And you, are you Nicola's brother?	A bit, not much

This amounts to an incomplete understanding of the (normative) properties of the relation brother of. Yet these properties, or their inverses, are true of all relations (Lemmon, 1966; Piaget, 1949).

- Reasoning about classes

Causal facts. Classification is a foundational ability, present in infancy. Classification in terms of class logic is an advance during childhood. Is this development due to equilibration, manifest as the interiorisation of action into operations? Or is it due to observational marking as a cue to a superordinate class? Chapman and McBride (1992) presented evidence in line with the cue-marking hypothesis.

Normative facts. Geneva is a French-speaking, international city and canton (county) situated to the east of France, so Swiss children have ample experience of citizens and foreigners. Their normative understanding of the vicariant classification (own-other nation) was investigated thus (Piaget, 1995a:251, 264):

What is Switzerland?	It's a country
And Geneva?	It's a city
Where is Geneva?	In Switzerland
Are you Swiss?	Yes
And are you Genevan?	Oh no, I'm already Swiss.....
What is your nationality?	I'm Swiss
Are you a foreigner in Swizerland?	No, I'm Swiss
And if you go to France?	I remain Swiss, the same as before
And a Frenchman who stays in France	It's the same as before, he remains a foreigner

Although the class Genevan is included in the class Swiss, this was denied by Claude who believed instead that these classes are exclusive: you can't be both. Claude does understand class membership, shown in his answer to the first three questions, but had misconceptions about class inclusion. Russell (1964) pointed out that it is major fallacy to confuse the (normative) relations of membership and inclusion in class logic. Ivan realised that the distinction between own-other nation is exclusive, but this amounted to a "false absolute". this is to miss the normative requirement of class logic that exhaustive dichotomies of the universal class are equivalent (Boole, 1958; Piaget, 1949).

Box 3 Five principle in *Recherche*

- action, organisation and identity

life is nothing but assimilation, the source of all organisation....the action of assimilation and that of undergoing the influence of context, or bring open to variation, or "imitating exterior factors" are two actions inversely related to each other. The better I assimilate, the more I remain self-identical. On the other hand, the more I vary, the less I am coherent, the less is my assimilating capacity and personality (p.155)

- action as the basis of knowledge

reason is a faculty born from action (p.50)

- actions as normative facts

action necessarily deforms the ideal in virtue of its mixture of fact and norm (*droit*) (p.116)

- normative facts as disequilibrium or equilibrium

fact is a form of equilibrium – or disequilibrium – and the ideal is another equilibrium, as real in a sense as the first, but often invoked rather than realised: the ideal is a limiting case, as the mathematicians say, or rather the full equilibrium onto which false or unstable equilibria of reality converge (p46)

- processes of equilibration

(i) *all organisation tends towards its own conservation (with two manifestations which are) equilibrium between an autonomous whole and its parts*
(ii) *only holistic equilibrium is the outcome of the form of that organisation, whilst partial equilibrium is a compromise between holistic equilibrium and the subsequent action of the ambiant context*
(iii) *all possible equilibria are nothing but combinations of these two types*
(iv) *all equilibria in living beings tends towards holistic equilibrium* (pp156-7)
Source: Piaget (1918)

Box 4 An epistemological quintet under an AEIOU mnemonic

Mary is asked the question $\boxed{2 + 2 = ?}$ to which her reply is $\boxed{2 + 2 = 4}$.
This is a correct response. But does it have – as well - any epistemological properties?

- autonomy: this equality holds in virtue of laws internal to the system of natural numbers in that 2 is a normative, not a causal, property of 4. This equality may have been compliantly accepted or accepted through reasoned assent (Piaget, 1971:49).
- equality (necessity): this equality is a necessary, not empirical, truth. There are no exceptions, because it could not be otherwise (Piaget, 1986:312). Some human knowledge is necessary knowledge.
- intersubjectivity: this equality is one and the same equality open to all, and so has the potential to be common ground between different thinkers (Piaget, 1995a:94)
- objectivity: this is a true equality, and so is not false. This difference is inherent in knowledge in that only truths can be known (Piaget, 1971:35)
- universality: this equality is always the case, and as such is one basis for mathematical generalisations such as the equivalence of addition and multiplication, $x + x = 2x$ (Piaget, 1968).

Source: Smith (1999b, c)

Box 5 Children's reasoning by mathematical induction

Reasoning by mathematical induction is a standard form of argument in mathematics and defined by Poincaré thus: if it is proved that a property is true of the number 1, and that it is true of $n + 1$ provided it is true of n, it is true of all whole numbers. This definition includes two criteria, one about a base property, and the other its generalisation (recurrence) in the number series. Frege and Russell accepted this definition, but argued that this reasoning was entirely logical in contradistinction to Poincaré's view who reckoned that such reasoning was partly logical and partly based on intuition.

In their study of children's reasoning by mathematical induction, Inhelder and Piaget (1963) sided with Poincaré. They drew two conclusions:

- children can reason by mathematical induction
- their reasoining is modal: if the induction is true, it is necessarily so

Their first conclusion was that children can successfully make a primitive form of this form of inference. This claim has gone largely unremarked. Their second conclusion was that children can understand the necessity of this inference. Necessary knowledge is modal knowledge which is reckoned to be under development in adolescence (Morris & Sloutsky, 1998; Moshman, 1998). Even so, the contrary view has also been stated that modal reasoning is under development during childhood (Piaget, 1986; Smith, 1999d).

Inhelder and Piaget's study was recently replicated. One hundred children aged 5-7 years in school Years 1 and 2 took part in the study and were individually interviewed twice. In the first interview (equal addition to equals), the children could see that two containers *A* and *B* were empty. In the second interview (equal addition to unequals), the children could see that *A* contained one counter but that *B* was empty. The children then repeatedly made equal additions to each box in three different contexts (open to observation; not open to observation; hypothetical addition, including generalised inference). The children were also asked modal questions about the necessity of their inferences. In each case, they were also invited to make a response and then to give reasons for it. There were three main findings:

- base criterion

Children in both school Years drew correct conclusions about the number in the containers. Across the three contexts in study I, 85% of the responses made by children in Year 2 were correct, which increased to 95% in the Year 2 children. The majority of these responses were justified.

- generalisation criterion

The children in both school Years made a generalised inference. They did so by detecting the ambiguity of *adding a great number*. Just over one third of the Year 1 children made a correct response over the two studies, and this increased to two-fifths for the Year 2 children. The companion question about *adding any number* was correctly answered by almost three-fifths of the Year 1 children over the two studies, and by almost nine-tenths of the Year 2 children. The majority of these responses were justified.

- modal criterion

There was an asymmetry about the children's modal reasoning. About two-fifths of the responses made by the Year 1 children were correct, and this increased (5%) for the Year 2 children. Only a minority (10%) of these modally correct responses were combined with a modal reason. Yet twice as many (20%) of the modally incorrect responses were justified by modal reasons. These included pseudo-modal reasoning (*that won't be fair on that one*); modality of action (*because you can do what number you want to*); modality of thought (*because if you had them all on your desk and your mother shouted, and then you just stood up and knocked them all off, some could have landed in the bin*).

The first two findings amount to good evidence Inhelder & Piaget's first conclusion. The third finding is partial evidence for their second conclusion. A possible mechanism of reasoning by mathematical induction is through iterative action which undergoes development during childhood.

Source: Smith (in press)

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